Biomechan Model Mechanobiol (2007) 6:189–197
DOI 10.1007/s10237-006-0054-x

ORIGINAL PAPER

A constitutive law for the failure behavior of medial collateral ligaments

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Received: 18 December 2005 / Accepted: 11 April 2006 / Published online: 25 August 2006

Abstract A constitutive model is proposed for the description of the tensile properties of medial collateral ligaments (MCLs). The model can reproduce the three regions – the toe region, the linear region, and the failure region – of the stress–stretch curve of ligamentous tissues. The collagen fibers are assumed to be the only load-bearing component of the tissues. They are all oriented along the physiological loading direction of the ligament. They are crimped in the slack configuration and are unable to sustain load. After becoming taut and before failing, each collagen fiber exhibits a linear elastic behavior. The fiber straightening and failure processes are defined stochastically by means of Weibull distributions. Published experimental data for the MCLs are employed to validate the constitutive relationship. Finally, the constitutive model is generalized in order to describe the three-dimensional mechanical behavior of the ligaments by following the structural approach.

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1 Introduction

Ligaments are connective tissues that consist of collagen and elastin embedded in a ground substance of water, proteoglycans, glycolipids, and fibroblasts. Collagen is the main load carrying component in ligamentous tissues. It is the most abundant protein constituting 65–80% of the ligament dry weight (Amiel et al. 1990). In parallel-fibered tissues, such as ligament, collagen is characterized by a hierarchal structure: collagen molecules are packed together to form collagen fibrils, collagen fibrils aggregate to form collagen fibers, and collagen fibers are arranged in fascicles that run parallel to the ligament loading direction (Kastelic et al. 1978).

Among the ligaments of the human body, the ligaments of the knee joint have been extensively studied by the biomechanics community due to the joint's elevated vulnerability. The medial collateral ligament (MCL) is the focus of the present study since it, together with the anterior cruciate ligament (ACL), is most prone to injuries (Bollen 2000). Injuries to the ligaments are classified according to their severity as first-, second-, and third-degree sprains. With a first-degree sprain, the ligament is only overstretched. A second-degree sprain occurs when the ligament is partially torn while a third-degree sprain consists of a complete rupture of the ligament. Understanding the mechanism of tearing in ligamentous materials is important for the prevention, the diagnosis, and the treatment of the injuries. Toward this end, experimental investigations complemented with reliable constitutive descriptions are needed to study the disruption of the ligamentous fibers associated with the injuries.

Progress in experimental mechanics has significantly contributed to characterize the mechanics of the ligaments. Nevertheless, difficulties persist in studying experimentally the failure behavior of the ligamentous tissue. Therefore, constitutive equations are needed to enhance our understanding of the mechanisms of ligamentous injuries and to guide the design of opportune experiments. Particularly, structural constitutive models can help in clarifying the relation between the biological architecture and the mechanical failure behavior of the tissues. These models are derived by modeling the tissue's components, their geometry, and their interactions and, hence, their material parameters are directly related to the tissue's structure. Although many structural models have been proposed for the description of the mechanics of collagenous tissues (Viidik 1969; Diamant et al. 1972; Stouffer et al. 1985; Comninou and Yannas 1976; Decraemer et al. 1980; Kastelic et al. 1980; Lanir 1979, 1983; Zioupos and Barbenel 1994; Humphrey and Yin 1997; Sacks 2000; De Vita and Slaughter 2006), only few models have been formulated to illustrate the failure process in these tissues (Kwan and Woo 1989; Hurschler et al. 1997; Liao and Belkoff 1999; Wren and Carter 1998).

To the authors' knowledge, the first attempt at modeling failure in parallel-fibered collagenous tissues was made by Kwan and Woo (1989). They developed a one-dimensional microstructural model in which collagen fibrils were assumed to be responsible for the gross nonlinear response of the tissue. In their model, the tissue was considered to be composed of finite groups of fibrils with different uncrimping strains and failure strains. The stress–strain relationship for the collagen fibril was assumed to be bilinear. The model was fitted to rabbit ACL and canine MCL experimental data but 11 parameters were needed.

The most complete theoretical description of failure for ligaments and tendons has been presented by Hurschler et al. (1997). Their constitutive law was formulated by modeling the ground substance, the collagen fiber and fibril structures. The tissue stress–stretch relationship was defined by considering the sequential uncrimping and stretching of collagen fibers. The fiber recruitment process was defined by a one-sided distribution. Interestingly, by following the structural approach developed by Lanir (1979) the constitutive law for an individual fiber was assumed to be determined by the fibril kinematics and spatial orientation. The ground substance, or matrix, was assumed to contribute to the gross mechanical behavior through a hydrostatic pressure term. Stretch-based failure criteria were introduced in the model at the fibril level for disorganized tissue and at the fiber level for parallel-fibered tissues. In both cases, the failure stretch was assumed to be equal for all fibrils or for all the straight fibers. The constitutive relation was simplified in order to curve fit failure experimental data of healing rat MCLs.

In a follow-up study, Liao and Belkoff (1999) presented a failure model for the tensile properties of ligaments that incorporates the gradual recruitment and stretching of collagen fibers. Differently from Hurschler et al. (1997), the fiber recruitment was described by a two-sided distribution and, therefore, some fibers could unrealistically become straight at a negative stretch. Each collagen fiber was assumed to be linear elastic and to fail at the same stretch in the taut configuration. This model has the merit of containing only four structural parameters. Although the model was found to describe well the abrupt failure behavior observed in experimental studies on rabbit MCLs, it could not reproduce the gradual failure behavior also observed in such studies.

Wren and Carter (1998) proposed a structural law for the tensile constitutive prefailure and failure behavior of soft skeletal connective tissues. The mathematical model accounted not only for the collagen fiber uncrimping, stretching, breakage, orientation, and volume fraction as the above-cited models (Kwan and Woo 1989; Hurschler et al. 1997; Liao and Belkoff 1999) but also for matrix constitutive behavior and its resistance to fiber reorientation. Experimental data from tendon, meniscus, and articular cartilage were used to validate the model. The values of the many structural parameters, which appear in the model, were extrapolated from different experimental data sets to simulate the experimental stress–strain curves.

In this study, a novel structural constitutive model is formulated to describe the tensile failure behavior of ligaments under the assumption that, after losing their waviness, the collagen fibers fail at different stretches. Five structural parameters are needed to reproduce the response of the ligaments exhibited during tensile tests. The model is validated by using published stress–strain data for the MCL, and it is compared to a similar constitutive model which is based on the common assumption that all straight fibers in the tissue fail at an identical stretch. Although a three-dimensional model is also proposed by following the approach proposed by Lanir (1979, 1983), it is not validated due to the lack of multiaxial histo-mechanical data for ligaments.

2 Model formulation

A one-dimensional model is first presented to describe the tensile prefailure and failure behavior of ligaments in Sect. 2.1. The ligament is idealized as composed of collagen fibers that are aligned along its direction of physiological loading. Elastin is assumed not to contribute to the mechanical behavior since its amount is not significant. The collagen fibers are assumed to be linear elastic and possess the same stiffness. They contribute to the ligament's mechanical response after becoming taut and before breaking. The fiber bending and compressive stiffnesses are ignored as well as fiber–fiber and matrix–fiber interactions. Moreover, viscous effects are not taken into account. The failure criterion is stretch based but, differently from previous studies (Hurschler et al. 1997; Wren and Carter 1998; Liao and Belkoff 1999), the taut fibers in the tissue are postulated to break at different stretches. Both the fiber straightening and fiber breakage are statistically defined by Weibull cumulative distributions. Subsequently, a general three-dimensional material law is proposed in Sect. 2.2 by following Lanir's pioneering work in soft tissue's structural constitutive modeling (Lanir 1979, 1983).

2.1 Recruitment and failure model

The mechanical response of MCL to tensile loading is assumed to be determined solely by the collagen component. In particular, the ligament is modeled by N parallel collagen fibers, where N is a positive integer that is large enough for the model to be statistically representative of the real system. The fibers are assumed to be all aligned along the main physiological loading direction of the ligament. They are characterized by having different straightening and failure stretches.

Let i be an integer with $i = 1, \ldots, N$. The generic collagen fiber i of the ligament has a straightening stretch, $\Lambda_s^{(i)}$, which is the stretch at which the fiber becomes straight, and a failure stretch, $\Lambda_f^{(i)}$, which is the stretch at which the fiber fails after becoming straight (see Fig. 1). The straightening and failure stretches for the N fibers are randomly defined according to Weibull distributions. They are numerically generated by transforming uniform deviates, which are random numbers between 0 and 1, into Weibull distributed random numbers by invoking the *fundamental transformation law of probabilities* as described in detail by Press et al. (1992). Thus, let $G_s^{(i)}$ and $G_f^{(i)}$ denote uniform deviates. The straightening and failure stretches of the collagen fibers are determined by using the following relationships

$$\begin{aligned} \Lambda_s^{(i)} &= 1 + \beta_s[-\ln(1 - G_s^{(i)})]^{1/\alpha_s} , \\ \Lambda_f^{(i)} &= 1 + \beta_f[-\ln(1 - G_f^{(i)})]^{1/\alpha_f} , \end{aligned} \tag{1}$$

where $\alpha_s > 0$ and $\beta_s > 0$ are the shape and scale parameters of the Weibull distribution describing the fiber straightening and $\alpha_f > 0$ and $\beta_f > 0$ are the shape and scale parameters of the Weibull distribution governing the fiber failure. It needs to be noted that the location parameters of the Weibull distributions (1) are set to be equal to 1 since the fibers are assumed to be all undulated in the reference configuration and not to break before becoming straight (see Fig. 2).

Let $\sigma^{(i)}$ denote the stress associated with a generic fiber i and let Λ denote the ligament's stretch. Then, $\Lambda/\Lambda_s^{(i)}$ is the fiber's stretch relative to the taut configuration. The stress for a generic fiber i is defined as follows

$$\sigma^{(i)}(\Lambda, \Lambda_s^{(i)}, \Lambda_f^{(i)}) = \begin{cases} 0 & \text{if } \frac{\Lambda}{\Lambda_s^{(i)}} \le 1 ; \\ K\left(\frac{\Lambda}{\Lambda_s^{(i)}} - 1\right) & \text{if } 1 < \frac{\Lambda}{\Lambda_s^{(i)}} < \Lambda_f^{(i)} ; \\ 0 & \text{if } \frac{\Lambda}{\Lambda_s^{(i)}} \ge \Lambda_f^{(i)} , \end{cases} \tag{2}$$

where K is the fiber stiffness that is equal for all fibers comprising the ligament. Relation (2) defines the constitutive relation for each fiber i. It states that the stress for the fiber i is zero when $\Lambda/\Lambda_s^{(i)} \le 1$. In this case, the fiber is undulated since the ligament's stretch is smaller than or equal to the fiber's straightening stretch. The stress for the fiber i is modeled as a linear elastic material when $1 < \Lambda/\Lambda_s^{(i)} < \Lambda_f^{(i)}$. In this case, the ligament's stretch is greater than the fiber's straightening stretch but smaller than the fiber's failure stretch. In other words, since the fiber i is taut when the ligament is stretched of the amount Λ, it produces stress before breaking. Finally, the stress for the fiber i is zero when $\Lambda/\Lambda_s^{(i)} \ge \Lambda_f^{(i)}$, i.e. when the straight fiber i exceeds its critical failure stretch. Moreover, it needs to be emphasized that the fiber can only rupture after losing its crimped morphology, i.e. when $\Lambda/\Lambda_s^{(i)} > 1$.

For $i = 1, \ldots, N$, the stress $\sigma^{(i)}$ of the fiber i with associate $\Lambda_s^{(i)}$ and $\Lambda_f^{(i)}$, is computed by means of relation (2). Then, the overall stress of the ligament, σ, is defined as the average of the stresses for the N collagen fibers by

$$\sigma(\Lambda) = \frac{1}{N} \sum_{i=1}^{N} \sigma^{(i)}(\Lambda, \Lambda_s^{(i)}, \Lambda_f^{(i)}) . \tag{3}$$

In conclusion, the uniaxial stress–stretch relationship $\sigma(\Lambda)$ for the ligament is numerically defined by Eqs. 1–3. Therefore, a set of five material parameters, $\{K, \alpha_s, \beta_s, \alpha_f, \beta_f\}$, needs to be determined by fitting experimental curves of σ versus Λ.

2.2 Model generalization

The one-dimensional model proposed in Sect. 2.1 can be generalized in order to describe the three-dimensional mechanical behavior of ligaments by using Lanir's structural approach for soft tissues (Lanir 1979, 1983). The

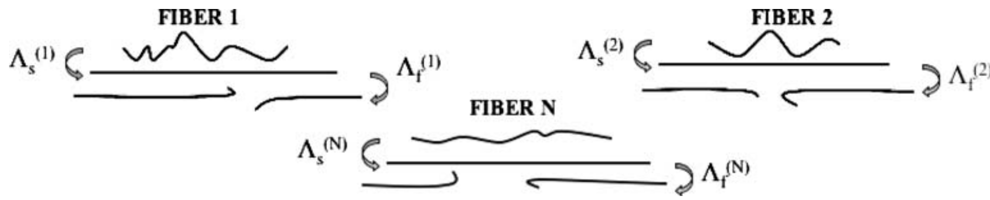
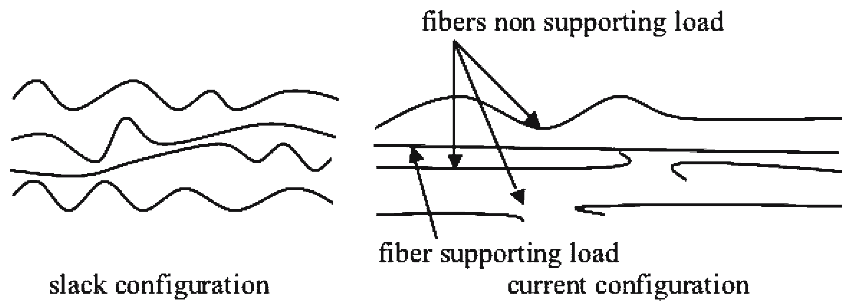


Fig. 1 $\Lambda_s^{(i)}$: straightening fiber stretch, $\Lambda_f^{(i)}$: failure fiber stretch

Fig. 2 Assumption of the recruitment and failure model



first Piola–Kirchhoff stress tensor $\boldsymbol{P}$ can be expressed as (Truesdell and Noll 1965)

$$\boldsymbol{P} = -p\boldsymbol{F}^{-\mathsf{T}} + 2\boldsymbol{F} \cdot \frac{\partial W(\boldsymbol{C})}{\partial \boldsymbol{C}} , \tag{4}$$

where '$\cdot$' denotes the dot product, p is an indeterminate pressure enforcing the incompressibility assumption, $\boldsymbol{F}$ is the deformation gradient tensor, $\boldsymbol{F}^{\mathsf{T}}$ and $\boldsymbol{F}^{-\mathsf{T}}$ are its transpose and inverse transpose, respectively, and $\boldsymbol{C} \equiv \boldsymbol{F}^{\mathsf{T}} \cdot \boldsymbol{F}$ is the right Cauchy–Green deformation tensor. The choice of $\boldsymbol{C}$ as a measure of the deformation guarantees that the principle of frame indifference is satisfied. $W = W(\boldsymbol{C})$ is the elastic potential that is defined as follows (Lanir 1979, 1983)

$$W(\boldsymbol{C}) = \int_{\Sigma} R(\hat{\boldsymbol{M}}) w(\Lambda(\boldsymbol{C}, \hat{\boldsymbol{M}}))\, \mathrm{d}\Sigma , \tag{5}$$

where Σ is the set of all material directions in the reference configuration, $\hat{\boldsymbol{M}}$ is an arbitrary material direction in Σ, $R(\hat{\boldsymbol{M}})$ is the probability density function for collagen fibers to be aligned in the direction $\hat{\boldsymbol{M}}$, and $w(\Lambda(\boldsymbol{C}, \hat{\boldsymbol{M}})$ is the elastic potential of collagen fibers in the direction $\hat{\boldsymbol{M}}$ determined by the axial fiber stretch

$$\Lambda(\boldsymbol{C}, \hat{\boldsymbol{M}}) = \sqrt{\hat{\boldsymbol{M}} \cdot \boldsymbol{C} \cdot \hat{\boldsymbol{M}}} . \tag{6}$$

According to Eq. 6, the stretch of each fiber Λ along its mean axis $\hat{\boldsymbol{M}}$ is derived from an affine transformation of the overall tissue's strain $\boldsymbol{C}$.

After defining the fiber elastic stress $\sigma(\Lambda)$ as

$$\sigma(\Lambda) \equiv \frac{\mathrm{d}w(\Lambda)}{\mathrm{d}\Lambda} , \tag{7}$$

the constitutive equation (4) takes the form

$$\boldsymbol{P} = -p\boldsymbol{F}^{-\mathsf{T}} + \boldsymbol{F} \cdot \int_{\Sigma} R(\hat{\boldsymbol{M}}) \frac{\hat{\boldsymbol{M}}\hat{\boldsymbol{M}}}{\Lambda(\boldsymbol{C}, \hat{\boldsymbol{M}})} \sigma(\Lambda(\boldsymbol{C}, \hat{\boldsymbol{M}}))\mathrm{d}\Sigma . \tag{8}$$

Given the collagen fiber distribution $R(\hat{\boldsymbol{M}})$ and the fiber stress–stretch relation $\sigma(\Lambda)$ defined by Eqs. 1–3, the constitutive law (8) defines the multiaxial mechanical response of the ligamentous material.

The three-dimensional model (8) can be reduced to the one-dimensional model (1)–(3) under certain assumptions. Assume that the ligament undergoes an isochoric axisymmetric deformation defined by the following deformation gradient

$$\boldsymbol{F} = \lambda^{-1/2} \boldsymbol{e}_r \boldsymbol{E}_R + \lambda^{-1/2} \boldsymbol{e}_\theta \boldsymbol{E}_\Theta + \lambda \boldsymbol{e}_z \boldsymbol{E}_Z , \tag{9}$$

where λ is the axial stretch, $\{\boldsymbol{E}_R, \boldsymbol{E}_\Theta, \boldsymbol{E}_Z\}$ and $\{\boldsymbol{e}_r, \boldsymbol{e}_\theta, \boldsymbol{e}_z\}$ are orthonormal bases such that $\boldsymbol{E}_Z$ and $\boldsymbol{e}_z$ are unit vectors parallel to the direction of physiological loading of the ligament in the reference and current configurations, respectively. Consequently, the right Cauchy–Green deformation tensor is given by

$$\boldsymbol{C} = \lambda^{-1} \boldsymbol{E}_R \boldsymbol{E}_R + \lambda^{-1} \boldsymbol{E}_\Theta \boldsymbol{E}_\Theta + \lambda^2 \boldsymbol{E}_Z \boldsymbol{E}_Z . \tag{10}$$

Moreover, assume that collagen fibers are perfectly parallel to the direction of loading so that $R(\hat{\boldsymbol{M}}) = \delta(\hat{\boldsymbol{M}} - \boldsymbol{E}_Z)$ where δ is the Dirac-delta function. It then follows from Eqs. 6–10 that the nonzero components of the first Piola–Kirchhoff stress are

$$P_{rR} = P_{\theta\Theta} = -p\lambda^{1/2} , \quad P_{zZ} = -p\lambda^{-1} + \sigma(\lambda) . \tag{11}$$

Because of the traction-free boundary condition on the lateral surface of the ligament, the indeterminate pressure term p assumes zero value. One then obtains that the only nonzero component of the stress, P_{zZ}, reduces to the axial fiber stress, σ, defined by Eqs. 1–3.

3 Results

The constitutive model defined by Eqs. 1–3 for the tensile behavior of MCLs has been numerically implemented in order to estimate its ability to reproduce experimental observations. Toward this end, the number N of the collagen fibers that form the ligament has been chosen to be equal to $10,000$ since no significant differences have been found by increasing such number. It needs to be noted that this number does not represent the effective number of fibers which occupy the ligamentous substance. Indeed, in the numerical implementation of the model, the number N can be increased without observing differences in the value of the stress σ defined by Eq. 3.

The random straightening and failure stretches of the collagen fibers, $\Lambda_s^{(i)}$ and $\Lambda_f^{(i)}$ given by Eq. 1, have been numerically generated by transforming the uniform deviates, $G_s^{(i)}$ and $G_f^{(i)}$, into Weibull distributed deviates as indicated in Press et al. (1992). The uniform deviates are computed by using Park and Miller's Minimal Standard generator with an additional shuffle (Park and Miller 1988). Subsequently, given the ligament's stretch Λ, the computation of the stress $\sigma^{(i)}$ for each fiber i, which is individuated by $\Lambda_s^{(i)}$ and $\Lambda_f^{(i)}$, is achieved by implementing relation (2). Once the stresses $\sigma^{(i)}$ with $i = 1, \ldots, N$ are obtained, the total stress of the ligament σ as a function of Λ is calculated by using Eq. 3. Moreover, the fractions of crimped, straight, and broken fibers at a fixed value of the tissue's stretch Λ can be readily estimated by placing counters in relation (2).

The set of material parameters $\{K, \alpha_s, \beta_s, \alpha_f, \beta_f\}$ that appear in the model has been identified by minimizing the sum of squares difference between experimental and theoretical stresses using the Downhill Simplex Method (Nelder and Mead 1965; Press et al. 1992). This method permits the evaluation of the minimum of a function with several independent variables without requiring the computation of its derivatives.

The MCL tensile test data published by Abramowitch et al. (2003) and by Provenzano et al. (2002a) have been employed to test the proposed constitutive model. Abramowitch et al. (2003) have performed uniaxial tensile tests on femur–MCL–tibia complexes to evaluate the goat as animal model for studying the MCL healing process. They have reported a typical stress–strain data that shows the MCL tearing and complete failure. These data are well fitted by the presented model as Fig. 3 illustrates. The model is able to describe the toe region, the linear region and, most importantly, the failure region of the stress–strain curve. The values of the parameters have been found to be $K = 460$ MPa, $\alpha_s = 1.74$, $\beta_s = 0.02$, $\alpha_f = 8.10$, and $\beta_s = 0.18$ ($R^2 = 0.99$). In Fig. 3, the fractions of taut fibers and the fractions of broken fibers are also depicted.

As mentioned earlier, a common assumption in previous works on modeling failure in soft tissues is that the fibers, which comprise the tissues, have an identical failure stretch, defined relatively to the taut configuration (Hurschler et al. 1997; Wren and Carter 1998; Liao and Belkoff 1999). By invoking this assumption, the single fiber stress takes the following form

$$\sigma^{(i)}(\Lambda, \Lambda_s^{(i)}, \Lambda_f^{(i)}) = \begin{cases} 0 & \text{if } \frac{\Lambda}{\Lambda_s^{(i)}} \leq 1\ ; \\ K\left(\frac{\Lambda}{\Lambda_s^{(i)}} - 1\right) & \text{if } 1 < \frac{\Lambda}{\Lambda_s^{(i)}} < \Lambda_f\ ; \\ 0 & \text{if } \frac{\Lambda}{\Lambda_s^{(i)}} \geq \Lambda_f\ , \end{cases} \tag{12}$$

where Λ_f is the failure fiber stretch with respect to the taut configuration and the other quantities appearing in Eq. 12 are defined as before (Sect. 2.1).

The proposed constitutive model is compared with a constitutive model defined by Eqs. 1, 3, and 12. It is noteworthy that, to simulate the disruption of the ligament, two parameters, α_f and β_f, are needed to randomly generate the failure fiber stretches, $\Lambda_f^{(i)}$, in the model (1)–(3) whereas one parameter, Λ_f, is needed to define the failure fiber stretch in the model (1), (3), and (12).

Figure 4 presents the comparison between the curve fittings of the models described by Eqs. 1–3 and by Eqs. 1, 3, and 12. The four best fitting parameters for the latter model have been determined to be $K = 716$ MPa, $\alpha_s = 0.89$, $\beta_s = 0.07$, $\Lambda_f = 1.16$ ($R^2 = 0.98$). As Fig. 4 shows, the newly proposed model can fit the data better than the four parameter model (1), (3), and (12).

Provenzano et al. (2002a) have conducted an experimental study to analyze the subfailure damage in ligament. In their study, they have subjected rat femur–MCL–tibia complexes to tensile tests in order to measure the mechanical properties of the ligament before and after applying subfailure stretches. A good agreement has been found between the proposed model and the experimental data obtained from one ligament–bone complex. Figure 5 displays the curve fitting of the model with the experimental data, the fractions of straight fibers and the fractions of broken fibers. The material

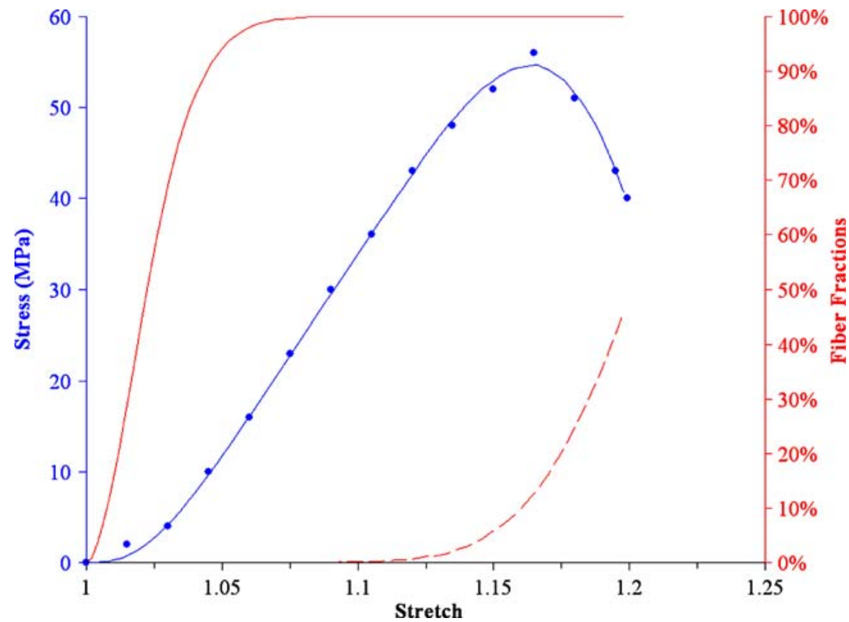


Fig. 3 Stress–strain experimental data from Abramowitch et al. (2003) (*blue filled circle*) with model fit (*blue continuous line*), fractions of straight fibers (*red continuous line*), and fractions of broken fibers (*red dashed line*)

parameters have been estimated to be $K = 1,345$ MPa, $\alpha_s = 1$, $\beta_s = 0.03$, $\alpha_f = 2.47$, and $\beta_s = 0.12$ ($R^2 = 0.99$).

The constitutive model (1), (3), and (12) has also been fitted to the experimental data published by Provenzano et al. (2002a). In Fig. 6, the curve fitting is compared with the curve fitting obtained by using the newly proposed constitutive law. The value of the four material parameters, $\{K, \alpha_s, \beta_s, \Lambda_f\}$, embodied in the model (1), (3), and (12) could not be uniquely determined. As evidenced by the results in Fig. 6, the proposed model reproduces better the MCL experimental stress–stretch curve.

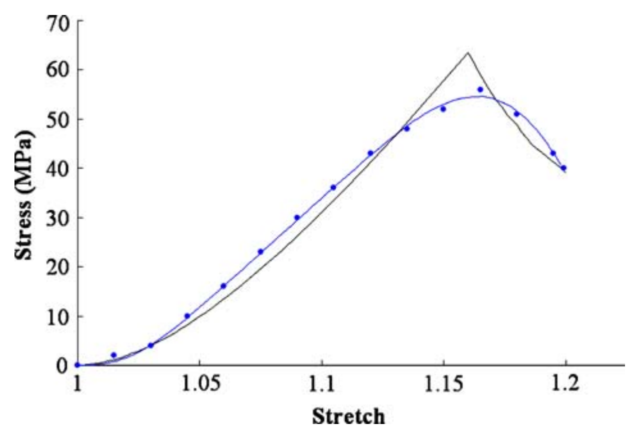


Fig. 4 Stress–strain experimental data from Abramowitch et al. (2003) (*blue filled circle*) with five parameter model fit (*blue continuous line*) and four parameter model fit (*continuous line*)

4 Discussion

A novel structural constitutive model for the description of the tensile behavior of knee ligaments is presented. The model is formulated by assuming that the ligament is composed of undulated collagen fibers that straighten out upon stretching. They are assumed to bear load only after losing their waviness and until they rupture. The recruitment and the disruption of collagen fibers are defined by statistical distributions. Differently from previous models (Hurschler et al. 1997; Wren and Carter 1998; Liao and Belkoff 1999), the straight fibers are assumed to fail at different failure stretches. The model is able to properly reproduce the toe region, the linear region, and the failure region of stress–strain curves of MCLs reported in published experimental studies (Abramowitch et al. 2003; Provenzano et al. 2002a). Furthermore, an extension to a three-dimensional material law is formulated within the context of structural mechanics for soft tissues (Lanir 1979, 1983).

The good agreement between the proposed constitutive model and the uniaxial experimental data published by Abramowitch et al. (2003) and Provenzano et al. (2002a) confirms the utility of the model in describing the process leading to partial and complete rupture of ligamentous tissues. The five parameters, which appear in the model, are sufficient to illustrate the tensile behavior of these tissues. Since the model is structurally based, these parameters provide insight into the relation between the histology and the mechanics of the tissues. The estimated stiffness constants of the collagen fiber for the goat and rat MCLs are within the stiffness range reported in the biomechanical literature (Sasaki and Odajima 1996; Fung 1993; Liao and Belkoff 1999). The remaining parameters permit the determination of

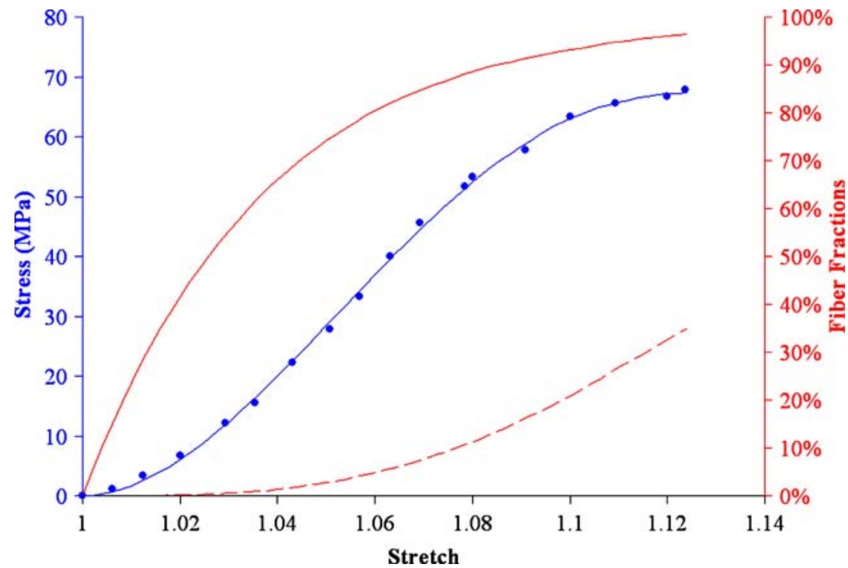


Fig. 5 Stress–strain experimental data from Provenzano et al. (2002a) (*blue filled circle*) with model fit (*blue continuous line*), fractions of straight fibers (*red continuous line*), and fractions of broken fibers (*red dashed line*)

the percentages of taut and broken fibers at each value of the tissue's stretch.

The tensile properties of the goat MCL are related to the recruitment and failure of collagen fibers in Fig. 3. It can be seen that the percentage of straight fibers increases gradually with stretch in the toe region of the stress–stretch curve. The fibers are taut and contribute to the overall tissue's stress in the linear region of the curve. Finally, the ligament is torn when 46% of the collagen fibers fail as shown in Fig. 3.

The rat MCL stress–stretch data reported by Provenzano et al. (2002a) are characterized by the absence of a distinct linear region (see Fig. 5). For this reason, the constitutive model predicts that the ligament experiences a complete rupture when 96% of the collagen fibers are recruited to bear load and 35% of such fibers fail. This suggests that some collagen fibers remain crimped when the ligament breaks.

The stress of the MCLs exhibited either an abrupt or a gradual drop in the failure region of the stress–strain experimental data. The cause of the different shapes of the stress–stretch curves is unclear and can be ascribed to numerous factors that include experimental methodologies and animal model age, species, and sex. Liao and Belkoff (1999) speculated that this difference in the failure regime is age related. In their study on rabbits, they found that the 4-month-old MCLs exhibit a prolonged failure region whereas the 7-month-old MCLs exhibit an abrupt failure region. It needs to be noted that the model presented herein is well suited to reproduce the MCL stress–stretch curves where the failure region appears to be gradual. However, when the MCL fails abruptly, the lack of a gradual failure region in the

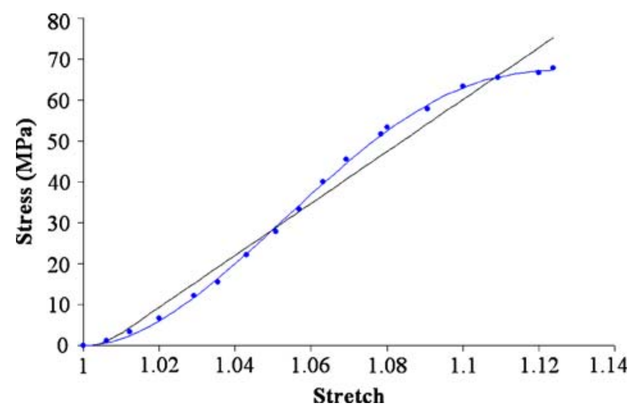


Fig. 6 Stress–strain experimental data from Provenzano et al. (2002a) (*blue filled circle*) with five parameter model fit (*blue continuous line*) and four parameter model fit (*continuous line*)

stress–stretch curve does not allow one to determine the parameters in the Weibull cumulative distribution describing the failure process.

The proposed model provides a better fit to the experimental data than the model formulated by assuming that the straight fibers in the ligamentous specimen have an identical failure stretch (see Figs. 4, 6). It needs to be noted that the model presented herein is akin to the model proposed by Wren and Carter (1998) in the definitions of fiber stress and the tissue's stress. In their formulation, soft tissues are viewed as composite materials in which both the fibers and the ground substance are assumed to contribute to the tissues' mechanical response. Moreover, these investigators introduced into their model the fiber volume fraction, the fiber orientation, and the resistance of the ground substance to the fiber reorientation. However, the values of the parameters in the model were inferred from different experimental studies in order to simulate the nonlinear stress–strain relationship of soft skeletal connective tissues.

Recent studies have revealed that the crimp period of collagen fibrils in rabbit MCLs is location dependent (Kukreti and Belkoff 2000). These inhomogeneities seem to suggest that the gross constitutive behavior of these ligaments must be derived by taking into consideration their fibrilar structure. Hurschler et al. (1997) developed a model for ligaments and tendons incorporating the structure of the tissues both at the fiber level and at the fibril level. However, their model could not be completely validated since the microstructural information, which is required for the determination of the material parameters, was not available.

In order to account for the anisotropic material behavior of MCLs, a three-dimensional model is also formulated. The one-dimensional model is generalized by adopting Lanir (1979, 1983) structural approach. The anisotropy of the tissue is modeled by introducing a statistical distribution for the collagen fiber orientation. However, the three-dimensional constitutive model is not validated since multiaxial mechanical tests complemented with quantification of the collagen fiber orientation are needed.

The MCLs exhibit short- and long-time memory effects that must be considered when modeling their mechanics during physiological activities. Recently, experimental studies have demonstrated that the quasi-linear viscoelastic theory proposed by Fung (1993), which has been widely used in biomechanics, is inadequate to describe the nonlinear properties of rat MCLs (Provenzano et al. 2001). To account for the experimentally observed nonlinear viscoelasticity of ligamentous tissues, Provenzano et al. (2002b) have proposed phenomenological models such as the nonlinear theory of Schapery (1969) and the modified superposition method (Findley et al. 1976). However, since the morphological changes that occur during creep and relaxation phenomena are different (Fung 1993; Thornton et al. 2000), it is believed that structurally based constitutive laws hold great promise in simulating the viscoelastic response of ligaments under different loading conditions. For this reason, in the future the proposed structural model will be modified to incorporate the description of the time- and history-dependent mechanical properties of ligaments.

While the model suggests that the collagen fiber alone is responsible for the mechanical behavior of the ligamentous tissue, it does not address other factors which may help to determine the failure properties of these tissues. The fluid-dominated ground substance influences the mechanics of the ligaments. However, little is known about its role in the failure mechanisms. Furthermore, because the stress–strain curves of the rat MCLs have been observed to change after a critical subfailure stretch along their directions of physiological loading (Provenzano et al. 2002a), it is speculated that damage of individual collagen fiber occurs during injury. A structural constitutive model, which accounts for damage of knee ligaments, will be the focus of future research.

Acknowledgements The authors would like to acknowledge Prof. Michael S. Sacks for valuable discussions on structural constitutive models for soft tissues. Funding was provided by NSF grant no. BES-9978858 and by a research grant from the Department of Mathematics, University of Naples II, Caserta, Italy.

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